PRACtiCE MidTerm #1)

Oscillation, Interaction

& Superposition

PHYSICS 204, MARTENS YAVERBAUM, ET AL.

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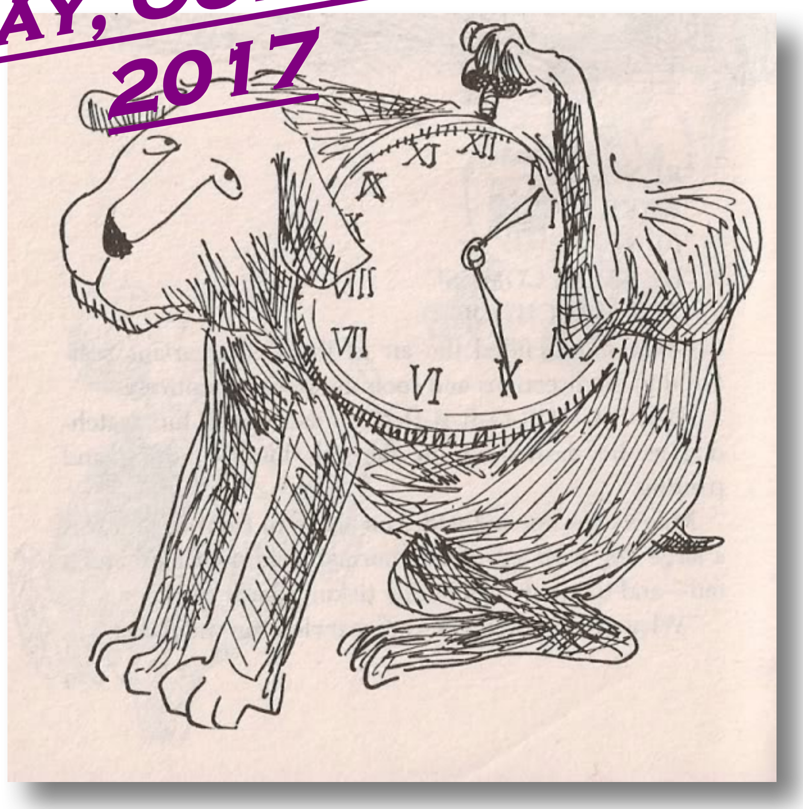

Name: ____

THE FOLLOWING RELATIONS UNDERLIE THE MATERIAL:

1) $\sum \vec{F} = m\vec{a}.$

2) $F = -Kx.$

3) $x = Acos(\omega t + \phi).$

4) $\omega \equiv 2\pi f.$

5) $f \equiv \frac{1}{T}.$

6) $\theta \equiv \frac{x}{r}.$

7) $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1.$

8) $\frac{\partial^2 y}{\partial t^2} = \left(\frac{T}{\mu}\right)\frac{\partial^2 y}{\partial x^2}.$

10) $y = Acos(\omega t - kx).$

11) $k \equiv \frac{2\pi}{\lambda}.$

12) $v = \frac{\omega}{k}.$

13) $\vec{F} = -\frac{GMm}{r^2}\hat{r}.$

14) $G \approx 6.67\ x\ 10^{-11}\,\frac{Nm^2}{kg^2}$

15) $\mu \equiv \lambda \equiv \frac{dm}{dx}.$

16) $\sigma \equiv \frac{dm}{dA}.$

17) $\rho \equiv \frac{dm}{dV}.$

1) MATH METHODS

a.

$$x = e^{-wt}$$

$$\frac{dx}{dt} = ?$$

$$\frac{d^2x}{dt^2} = ?$$

b. Differential Equations.

i. First, Just Assume that the following function is true and somehow useful.

$$x = A\cos(\omega t)$$

Then find:

$$\frac{dx}{dt} = ?$$

$$\frac{d^2x}{dt^2} = ?$$

ii. **Now switch gears:**

is $x = A\cos(\omega t)$ **'a solution'** *to*

$$\frac{d^2x}{dt^2} = -(\omega^2)x?$$

Why or why not?

iii. Now assume: $x = Acos(5t)$

Is x = A cos (5t) 'a solution' to

$\frac{d^2x}{dt^2}$ = -3x ?

Could there be conditions under which $x = A\cos(\omega t)$ *is* 'a solution' *to*

$$\frac{d^2x}{dt^2} = -(\frac{K}{m})x?$$

What would these conditions be?

Is $x = e^{-wt}$ a 'solution' to $\frac{d^2x}{dt^2} = -(\omega^2)x$?

Why or why not?

Assume that $i \equiv \sqrt{-1}$.

and Let $x = e^{-i\omega t}$.

$\frac{dx}{dt} = ?$

$\frac{d^2x}{dt^2} = ?$

Is $x = e^{-i\omega t}$ a 'solution' to $\frac{d^2x}{dt^2} = -(\omega^2)x$?

Why or why not?

2) SHO

A. **Spring**

Consider two possible descriptions for the behavior of some system, below:

i)

$$\frac{\partial^2 J^3}{\partial t^2} = -(LC)J^4;$$
$$LC = 3.14 \times 10^{-5}\,\sec^{-2};$$
$$J^5{}_0 = 300\; milli - Amperes^4.$$

and

ii)

$$\frac{\partial^2 p}{\partial t^2} = -(\beta^4)p;$$
$$\beta^4 = 3.14 \times 10^5\,rad^2/\sec^2;$$
$$p_0 = 300\; Pascals$$

a. Assuming that in each one, ***t*** stands for time
(as measured from some initial moment called 0),

indicate WHICH ONE of the above descriptions could refer to a system behaving in Simple Harmonic Oscillation; simply reply **i** or **ii** (3 pts).

b. For whichever expression you DID CHOOSE to be an SHO, find its ***period*** of oscillation; provide a *number*, measured in seconds (5 pts).

c. For whichever expression you DID CHOOSE to be an SHO, determine the value of the dependent variable at t = 100 seconds from the beginning of measurement. In your final answer, include whatever units are appropriate – according to the facts of that problem (5 pts).

B. **Fall.** (35 pts).

a. (includes all four parts below).

Consider the following differential equation:

$$\frac{d^2r}{dt^2} = -\left(\frac{Qq}{4\pi\varepsilon_0 mR^3}\right)r,$$

in which $\boldsymbol{Q, q, R, m}$ and $\boldsymbol{\varepsilon_0}$ are all non-zero constants,
$\boldsymbol{r}$ is a variable measured in meters
and $\boldsymbol{t}$ is a variable measured in seconds.

i. Show that $\boldsymbol{r = \frac{1}{2}at^2 + v_0 t}$ (in which $\boldsymbol{a}$ and $\boldsymbol{v_0}$ are constants; $\boldsymbol{a \neq 0 \neq v_0}$) ***IS NOT a solution*** to the differential equation presented above.

Your response must be largely *mathematical* yet include at least one clear thought expressed in *English* (7 pts).

ii. Show that $\boldsymbol{r} = \boldsymbol{r_0}\, cos(\omega t + \kappa\chi)$ (in which $\boldsymbol{\omega, \kappa, r_0}$ ***and*** $\boldsymbol{\chi}$ are <u>constants</u>) ***IS a solution*** to the *differential equation* above (4 pts).

iii. according to this solution, how is $\boldsymbol{\omega}$ *related to* the *constant* term(s) in the DIFFERENTIAL EQUATION (4 pts)?

a. according to this solution, in what units must the constant product $\boldsymbol{\kappa\chi}$ be measured (3 pts)?

Assume $\boldsymbol{\kappa\chi = 0, r_0 = 1}$ and $\frac{Qq}{4\pi\varepsilon_0 mR^3} \approx 9 \times 10^9$ s^{-2}

.

b. In seconds, what is the ***period of oscillation*** for the particle described by the above differential equation (7 pts)?

c. find the approximate value for ***r*** when $\boldsymbol{t = 2}$ ***SECONDS*** (7 pts)? CAREFUL.

d. If the value of the constant $\boldsymbol{\varepsilon_0}$ were to triple (while all else remained fixed), then ***what would happen*** to the ***number of oscillations*** we would expect to count ***per hour***? CAREFUL.
(Would the number increase or decrease; by **precisely how much**?) (3 pts.)

b. (includes all four parts below) (35 pts).

Near the surface of Earth, a particle of mass $\boldsymbol{m}$ is dangled from a long string, length $\boldsymbol{L}$; the particle oscillates along a small arc according to the differential equation

$$\frac{\partial^2 \theta}{\partial t^2} = -(20)\theta,$$

where $\boldsymbol{\theta}$ refers to an angular displacement measured from the vertical and $\boldsymbol{t}$ refers to time.

The particle's mass is given by $m = 8\ kg$.

The length of the string, L, is constant and was accidentally not recorded by the researchers – but can be deduced from all the other given information.

Whenever the particle arrives at a location of $\boldsymbol{\theta = .2\ radians}$ from the vertical, the particle has no instantaneous speed. On both sides of the vertical, that is, $\boldsymbol{\theta = .2\ radians}$ is repeatedly observed to be a 'turning point' for the particle's periodic motion.

i. Draw a clear diagram of this particle at some arbitrary point during oscillation, making sure to label variables and constants described above (4 pts).

ii. In what ***units*** should the constant (20) be measured (3 pts)?

iii. Approximating to three significant digits,
what is the ***angular frequency*** of this oscillator on a string (4 pts)?

iv. ***What is the particle's approximate SPEED at*** $t = 3$ ***seconds*** (4 pts)?

v. Assume that an experimenter begins measuring time at the instant the particle reaches .2 radians from the vertical. Assume, further, that the only force doing work on this dangling particle is gravity.

(a) ***How much Potential Energy*** does the particle have at $t = 0$ – ***immediately at the start*** (10 pts)?

(b) Assume that all the above values remain precisely the same, EXCEPT that the ***turning*** points at +/- .2 radians now occur at +/- .6 radians: In other words, the amplitude has been TRIPLED.
Given this particular change, describe what will happen to the value of the ***period*** of time for one cycle (10 pts)

6) THE PUDDING

1) FIRST, PROVIDE A CLEARLY LABELED DIAGRAM which presents the situation, clarifies many of the situational 'givens', and defines constants and variables to be mentioned in your derivation.

2) THEN with fundamental principles of physics, geometry, calculus and English,

DERIVE THE FOLLOWING STATEMENT you choose. That is, show how the statement follows from a series of steps – each of which can be justified as a small and reasonable inference from some step before.

To prove: ***"The motion of a small-angle, planar pendulum is approximately Simple Harmonic".***

For this derivation, You May ASSUME:

1) A particle of m is suspended at the end of a long, light string, length L;

2) It is held still at a small angular displacement from an equilibrium position – found at the lowest possible point on the vertical;

3) the small angular displacement is called θ.

4) It is then released – with no starting speed at all – from the small angular displacement, called θ.

5) You may also assume: a) Newton's 2nd Law,
b) The Small-Angle (Radian) Approximation for the Sine of an Angle,
c) Anything we have already established about the solution to the 2nd order Dif. Eq. for SHO.

(So please first write all the above into your sheet as "Givens"…)