$$\sum \vec{\mathbf{F}} = m\vec{a}$$

$$\vec{\mathbf{F}} = -k\vec{x}$$

$$v_0 = 0, \quad x_0 = A, \quad A \neq 0$$

$$x = f(t)$$

$$x = ??$$

$$-k\vec{x} = m\vec{a}$$

$$-kx = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$-\left(\frac{k}{m}\right)x = \frac{d^2x}{dt^2}$$

$$\boxed{\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x}$$

$$x = \ldots??$$

try: $x = \frac{1}{2}at^2 + v_0 t$

$$\frac{d^2x}{dt^2} \overset{???}{=\!=} -\left(\frac{k}{m}\right)x$$

$$\frac{dx}{dt} = at + v_0$$

$$\frac{d^2x}{dt^2} = a$$

$$a \overset{???}{=\!=} -\left(\frac{k}{m}\right)x$$

NO!

$if\ a = -\left(\frac{k}{m}\right)x,$ **then**

$x = -\left(\frac{m}{k}\right)a;$ **then**

$x = x_0;$ **then**

$\frac{dx}{dt} = \frac{d^2x}{dt^2} = 0;$ **but** $a \equiv \frac{d^2x}{dt^2}$

then $a = 0;$ **but** $F = ma = -kx,$

then $x = 0;$ **but** $x = x_0;$

then $x_0 = 0.$ *But* $x_0 \neq 0.$ *So... NO!*

$$\textit{try:} \quad x = \cos t$$

$$\frac{d^2x}{dt^2} \overset{???}{=\!=} -\left(\frac{k}{m}\right)x$$

$$\frac{dx}{dt} = -\sin t$$

$$\frac{d^2x}{dt^2} = -\cos t$$

$$-\cos t \overset{???}{=\!=} -\left(\frac{k}{m}\right)x$$

$$-\cos t \overset{???}{=\!=} -\left(\frac{k}{m}\right)\cos t$$

ok...

$$\textit{iff} \quad 1 = \frac{k}{m} \ldots$$

$$\textit{if } k = m \, (?!)$$

So... BAD !!!

$$try\ x = A\cos\omega t$$

$$\frac{d^2x}{dt^2} \stackrel{???}{=} -\left(\frac{k}{m}\right)x$$

$$\frac{dx}{dt} = -A\omega\sin\omega t$$

$$\frac{d^2x}{dt^2} = -A\omega^2\cos\omega t$$

$$\frac{d^2x}{dt^2} = -\omega^2 A\cos\omega t$$

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad !$$ **SO (yes!)**

$$\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x$$

IFF $\omega^2 = \frac{k}{m}$!!!

so if $\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x$

or $\frac{d^2z}{dt^2} = -\left(\frac{k}{m}\right)z$ or $\frac{d^2j}{dt^2} = -(\text{'constant'})j$ etc.,

then we have S.H.O. with known and constant ω:

$$j = A\cos\omega t$$

$$\omega = \sqrt{\text{'constant'}}$$

$$\mathbf{f} = \frac{\omega}{\mathbf{2}\pi}$$

$$\mathbf{T} = \frac{\mathbf{1}}{\mathbf{f}}$$

$$\mathbf{T} = \frac{\mathbf{2}\pi}{\omega}.$$