GUIDE TO QUESTION 2

2. The Inclined Plane is Moving!

A child seated in a car places a quarter on a clipboard. The coefficient of static friction between the clipboard and the quarter is 0.1. The car begins to accelerate forwards at a constant rate of 2 m/s^2. At the same time, the child tilts the clipboard forward (i.e. front end down, back end up).

GOAL: Find the range of angles (measured from the horizontal) that the child can tilt the clipboard if she wants the quarter not to move?

Notice that you have been given the ***ultimate GOAL*** of the problem at the beginning. All the individual steps are just there to ***help you reach that goal***. Try to keep this goal in the back of your mind as you work through each step below.

Your goal here is to find a ***range***. That's because there are many angles that would work. In fact, there are ***infinite*** angles. But if you want to find a range, you only need ***two*** values. ***Which two?***

A. Draw a pictorial diagram of the situation, including ***all known and unknown quantities***.
   What is going on in this scenario? What features & quantities are important? Label them.

B. If we are looking at the ***minimum*** angle, what direction will friction have to point to keep the quarter in place?
   To find the direction of a force, ask yourself: what would happen to the object of interest if this force were not there?

   If the clipboard were tilted at a very low angle (i.e. almost flat), and there were no friction, which way would the quarter tend to move, when the car accelerates?

C. If we are looking at the ***maximum*** angle, what direction will friction have to point to keep the quarter in place?

D. If the angle goes below the minimum angle, what will happen to static friction and what will happen to the quarter?
   This is a qualitative question. We don't want any numbers, just a description.

   But notice that the question has an "and" in it. Don't forget to answer both parts.

   The point of this question & the next is to help understand and explain an important and at-first surprising point: we will try to find the maximum & the minimum angle, but ***both*** of these will occur when static friction is at its ***maximum.*** See how that's weird?

   (Remember, the *minimum* f_s is always 0; i.e. when f_s is at its minimum, there ***is no*** f_s. That happens at the middle of the range.)

E. If the angle goes above the minimum angle, what will happen to static friction and what will happen to the quarter?

F. Draw a system schema of the quarter.

Find the ***minimum*** angle:
(this is just a heading: all steps G-M are about finding the minimum angle.)

G. Draw a ***pure*** FBD of the quarter for this scenario (minimum angle).
H. Choose a coordinate system in which the x-axis is parallel to the direction of acceleration (assuming that the quarter does ***not*** slide).
   "Direction of acceleration" means the direction of acceleration *of the object of interest*: the quarter. Remember *Galileo's Principle of Relativity*: some things are relative—and something things are ***not***.

I. Draw a ***component*** FBD of the quarter.
   Be careful. Your coordinate system determines which forces are considered "diagonal" and so must be broken up into components.

J. Write NII equations for the quarter on both axes.
K. Express $\boldsymbol{f}_{s(max)}$ in terms of **N**.
   This step is easy—just apply the equation. But there's an unwritten step here that you have to figure out:

   Since you know $\boldsymbol{f}_{s(max)}$ in terms of **N,** and you know that, at the min. angle, static friction is at its maximum (see hints under step D above), you can then substitute into your NII equations in a way that removes one of the unknowns.

L. Solve for **N** in terms of other variables in both equations.
   Hint 1: The quarter is ***not*** sliding.
   Hint 2: ***Do*** plug in 10 for ***g***.
   I.e. use algebra to rearrange each equation so that **N** is isolated on one side.

   But why? We're not actually looking for **N**, so why is this a good move?

M. Solve to find $\boldsymbol{\theta}_{min}$.
   I.e. find the minimum angle. This is "just" algebra.

Find the ***maximum*** angle.

N. Solve to find $\boldsymbol{\theta}_{max}$, using the same steps (G-M) that you used to find $\boldsymbol{\theta}_{min}$.
   Only one of the forces is different in this scenario—which one?
   This difference changes each & every step—but only a little bit.

   You've now found two angles. Did you reach the ***Ultimate Goal*** stated at the beginning? Did you find the ***range of angles***? Actually, you did. State your final answer ***as a range***.

NOTE: In addition to the version of this problem that you see here, you should be prepared for the "Rough Accelerating Plane" problem from the NII.3 homework (problem IV). You should know how to solve BOTH that problem and this one.