Lab 4: A Ballistic Projectile
Formal Report

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Physics 203-01 L1

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Abstract

The purpose of this experiment was to make a prediction about the past by re-enacting the scenario of a crime, by ~~determining~~ the location/horizontal displacement of a weapon (ballistic projectile) that was fired at a target (cup). Two photogates were used in the first setup of the ballistic projectile apparatus to measure the time it took for a bullet to travel between the beams of each gate. Five trials were conducted using the opening of Gate 1 followed by the opening of Gate 2 as the time interval for the bullet's flight. The displacement between the two photogates was measured to 19.4 +/- 0.05 cm. After finding the time and displacement, the horizontal velocity of the bullet was calculated for all five trials (using the definition of average velocity) and averaged to 602.6 cm/s, uncertainty interval (601.0, 604.1 cm/s). Using the measured height of the center of the bullet from the floor (89.1 +/- 0.1 cm), it was possible to calculate the amount of time it would take the bullet to fall to the floor with an initial velocity of 0 cm/s (0.464 s). The horizontal displacement of the bullet was calculated to 256.7 cm using an equation derived from the definitions of average acceleration and average velocity. To determine if the prediction was correct, the crime scenario was re-enacted by positioning the ballistic pendulum on a tabletop at a distance measured to 256.7 +/- 0.15 cm from the estimated center of a cup placed at an arbitrary distance from the table on the floor. The "weapon" was fired three times. In the first and second trials, the bullet grazed the inside edge of the cup. After adjusting the position of the weapon so that it was more accurately aligned with the center of the cup, the bullet landed in the center in the third trial. The horizontal displacement required for the bullet shot from the ballistic pendulum to hit the cup was determined to be 256.7 cm +/- 0.15 cm.

Introduction

A projectile is an object is that is fired or thrown into the air. Aristotle believed that a projectile thrown horizontally moved along a straight line mid-air until the projectile dropped to the ground. However, Galileo later discovered that projectiles thrown horizontally moved along a curved path. Galileo also stated that the horizontal and vertical components of this motion were independent from one another. By analyzing the horizontal and vertical velocity of a projectile separately, as in a grid system with x-y axes, it is possible to calculate the total displacement in either direction along an axis. In the Ballistic Projectile Experiment, the separate vertical and horizontal components of a bullet's velocity were explored in determining if it was possible to make a retrodiction about a crime scene - in other words, the location of a weapon relative to a target that had been struck.

Research Question

How can a retrodiction be made about the location of a weapon relative to a target? What is the horizontal displacement required for a bullet shot from a ballistic pendulum (placed on a table) to hit a target (cup placed on the floor)?

Data Collection

The Ballistic Projectile Experiment required determining where a "weapon" (ballistic pendulum) had been located if a "bullet" (ballistic projectile) were to hit a particular target (styrofoam cup). Setting up a model was the first step. A Cenco Ballistic Pendulum functioned as the weapon. Two photogates connected to a LabPro sensor interface, which was connected to a laptop containing LoggerPro 3.8.4 software, were mounted on pendulum using masking tape. The displacement between the two proximal edges of the photogates to the bullet was measured to 19.4 +/- 0.05 cm. A solid brass sphere 2.5 +/- 0.05 cm in diameter (with a hole bored in the center so it could be mounted on the pendulum) functioned as the bullet. Within the photogates were electrical circuits that recorded the time the bullet passed through and interrupted the circuit (specifically when the circuit was broken, or the gate was "opened," and when the circuit was re-formed, or when the gate was "closed). The software functioned as a digital stopwatch so the time it took the bullet to pass between two gates could be determined.

This was a two-day experiment. On the first day, the ballistic pendulum was fired and its time between photogates was recorded in five trials. Using these data in conjunction with the displacement between photogates, the horizontal velocity of the bullet as it passed between the gates was calculated to an average of 602.6 cm/s, uncertainty interval (601.0, 604.1 cm/s). The vertical displacement between the center of the bullet and the floor was measured to 89.1 +/- 0.1 cm.

On the second day, the data collected from the first day was used to derive the horizontal displacement of the bullet's path after being fired, which was calculated to 257.6 cm. The target in this experiment was a styrofoam cup that was taped to the floor at an angle and arbitrary distance from the pendulum. Using a meter stick to measure out the predicted distance between the cup and pendulum resulted in a displacement of 257.6 +/- 0.15 cm (combining three different measurements: from the estimated center of the cup to the edge of the table; from the edge of the tabletop to the surface of the bullet; and from the surface of the bullet to the center of the bullet). The pendulum was aligned with the cup and fired in three different trials. In the first trial, the bullet grazed the inside edge of the cup. The pendulum was readjusted to be more accurately aligned with the cup. In trial 2, the inside edge of the cup was grazed in a slightly lower position (attributed to moving the pendulum backwards slightly while loading it). In trial 3, the pendulum was again readjusted by replacing it in its original position while accurately aligning it with the cup; the bullet then landed in the center of the cup. In each trial, the bullet quickly bounced out of the cup after landing inside it.

Diagram - see following page

DIAGRAM 1: BALLISTIC PENDULUM

Wowza! +1

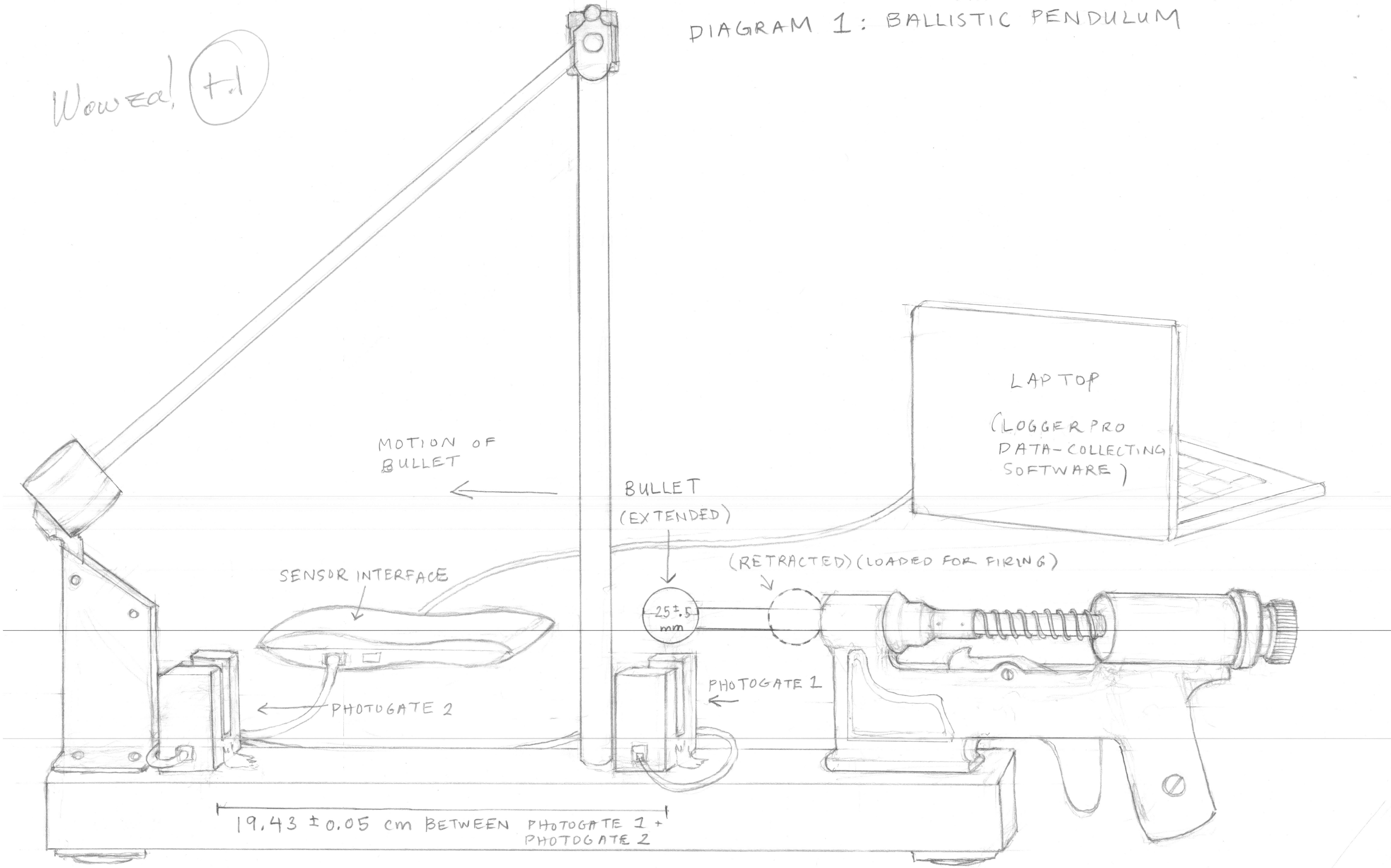

DIAGRAM BY MARGARET FARMER

DIAGRAM 2: RETROANALYSIS
(POSITIONING BULLET TO HIT TARGET)

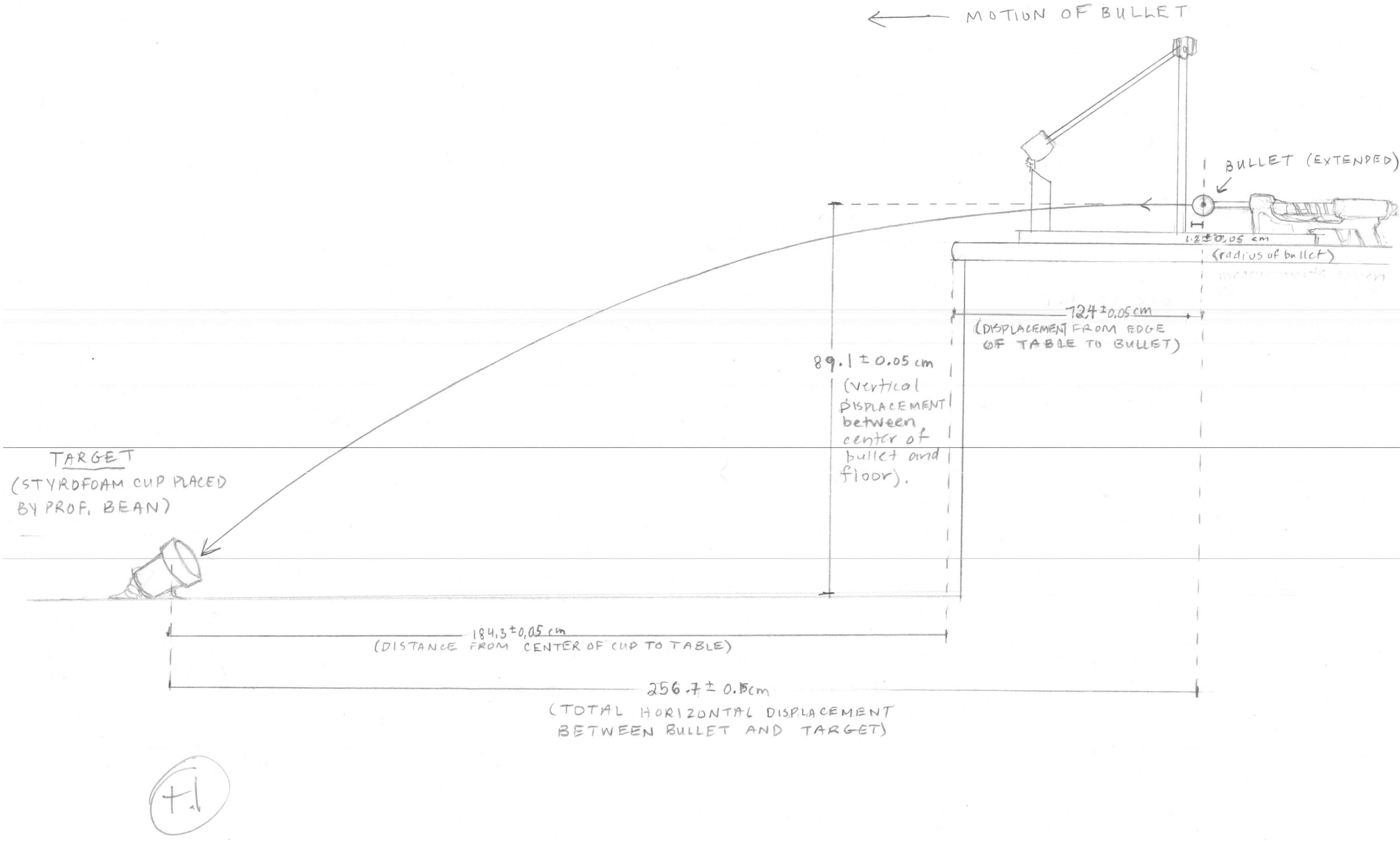

DIAGRAM BY MARGARET FARMER

Analysis

The path of a projectile can be separated into horizontal and vertical components, which can then be analyzed separately to predict the projectile's position a certain amount of time after it is fired into the air (see Appendix C for a list of equations used).

The horizontal velocity of the bullet was calculated first. The time elapsed between photogates during the bullet's flight was calculated by subtracting the time recorded by the software of the opening of Gate 1 from the opening of Gate 2. For the following example, the value from Trial 1 was used (0.032394 +/- 0.000001 s). See Appendix A for values from each trial. This process was repeated for each of the 5 trials. Using the measured displacement of 19.4 +/- 0.05 cm between photogates (taken from the edges of Gate 1 and Gate 2 proximal to the bullet), the horizontal velocity of the bullet was calculated using the following method:

$$V_x = \frac{\Delta x}{t}$$

Where V_x is the average horizontal velocity (cm/s) of the bullet when it passed between photogates; Δx is the change in position (in cm) of the bullet between the two photogates; and t is the amount of time elapsed (in seconds) between Gate 1 and Gate 2 during the bullet's flight. For example,

$$V_x = \frac{19.4\ cm}{0.032394s} = 598.9\ m/s$$

Repeating this calculation for all 5 trials (see Appendix for all calculations), the average horizontal velocity of the bullet when it was fired was calculated to be 602.6 cm/s.

Assuming the horizontal velocity of a projectile is constant (in other words, it does not accelerate horizontally), the following can also be assumed:

$$V_{0x} = V_x$$

Where V_{0x} is the initial horizontal velocity of the bullet when it was fired and V_x is the horizontal velocity of the bullet when it completed its trajectory and hit the floor.

Because the goal of the lab included finding a way to predict the bullet's horizontal displacement after being fired, it was necessary to calculate the time interval between firing and landing. Assuming the initial vertical velocity of the bullet was 0 cm/s, because it was fired at a purely horizontal angle (0°), the downward motion of the bullet was only affected by gravity (assuming negligible air resistance), which is the constant acceleration of -980 cm/s^2. Using the measured height of the center of the bullet (loaded on the pendulum) from the floor ([87.9 +/- 0.05] cm table height + [1.2 +/- 0.05] cm bullet = 89.1 +/- 0.1 cm), the following method was employed:

$$y - y_0 = \tfrac{1}{2}at^2 + V_{0y}t$$

Where y_0 is the initial vertical position of the bullet; y is the final vertical position of the bullet; a is the constant acceleration of the bullet downward due to gravity; t is the amount of time elapsed during the bullet's fall to the floor, and V_{0y} is the initial downward velocity of the bullet.

Substituting 89.1 cm for y_0, 0 cm/s for V_{0y}, and -980 cm/s² for *a*, the following value was derived:

$$0 - 81.9\ cm = \tfrac{1}{2}(-980\ cm/s^2)t^2 + (0\ cm/s)t$$

therefore

$$-89.1\ cm = -490 cm/s^2 t^2$$

$$t^2 = 0.1818\ s^2$$

$$t = 0.4264\ s \approx 0.426\ s$$

Next, to find the horizontal displacement of the bullet in 0.426 seconds, the following equation was used:

$$x - x_0 = \tfrac{1}{2}at^2 + V_{0x}t$$

Where x_0 is the initial position of the bullet on the pendulum; x is the position of the bullet when it hit the target; a is acceleration due to gravity, t is the time elapsed between the bullet being fired and the bullet hitting the target; and V_{0x} is the initial horizontal velocity of the bullet. x-x_0 is the total horizontal displacement of the bullet.

Because the horizontal velocity of the bullet was constant , a=0. Substituting 0.426 s for t and 602.6 cm/s for V_{0x}, and $X_0 = 0$, the equation can be written as follows:

$$x = \tfrac{1}{2}(0\ m/s^2)(0.426\ s)^2 + (602.6\ cm/s)(0.426\ s)$$

$$x = 256.7\ cm$$

The horizontal displacement of the bullet from the target was therefore calculated to be 256.7 cm. This value made a retrodiction possible for determining the horizontal displacement of a bullet fired horizontally from a height of 89.1 +/- 0.1 cm at a velocity of 602.6 cm/s. When the styrofoam cup (target) was placed at an arbitrary location on the floor, the ballistic pendulum was placed at a certain position on the table so that the bullet was able to accurately hit the target when fired.

Error and Uncertainty

The Absolute Uncertainty method was used to calculate uncertainty intervals for all measurements.

The horizontal displacement between the two photogates was measured to 19.4 +/- 0.05 cm using a standard meter stick (the smallest unit of measurement was 0.1 cm). The amount of time it took for the bullet to travel between the two photogates was determined by subtracting the time Photogate 1 opened from the time Photogate 2 opened (measured to 0.000001 +/- 0.0000005 seconds). When subtracting the value for time, the uncertainty intervals were added, giving a total uncertainty of +/- 0.000001 seconds.

When calculating the uncertainty intervals for the horizontal velocity of the bullet, both the distance uncertainty and time uncertainty had to be taken into account. To calculate the velocity uncertainty interval, the following method was used for each trial:

$$maximum\ uncertainty = \frac{max\ \Delta x}{min\ t}$$
$$minimum\ uncertainty = \frac{min\ \Delta x}{max\ t}$$

For example, in trial 1:

$$maximum\ velocity = \frac{19.45\ cm}{0.032393\ s} = 600.4\ cm/s$$
$$minimum\ velocity = \frac{19.35\ cm}{.032395\ s} = 597.4\ cm/s$$

The same procedure was used for each trial; all maximum values were then averaged, and all minimum values were averaged. The velocity of the bullet was therefore calculated to be 602.6 cm/s, uncertainty interval (604.1, 601.0) cm/s. See Appendix B for all maximum and minimum values.

The target was placed on the floor at an arbitrary position. When measuring the horizontal displacement, calculated earlier, between the target and the bullet, two displacements were measured out using meter sticks: the distance between the estimated center of the target on the floor and the edge of the table on which the pendulum sat (184.3 +/- 0.05 cm), and the distance between the edge of the table and the center of the bullet. The diameter of the bullet was measured to 2.5 +/- 0.05 cm with calipers and the radius was estimated to 1.25 +/- 0.05 cm; this value was incorporated into the horizontal displacement measured out on the tabletop - a total of 72.4 +/- 0.1 cm. These values added together totaled 256.7 +/- 0.15 cm. The same method was used for calculating the height of the center of the bullet from the floor ([87.9 +/- 0.05] cm + [1.2 +/- 0.05] cm = 89.1 +/- 0.1 cm) The bullet itself was on the pendulum in the extended position when measurements were taken (see diagram).

The ballistic pendulum was fired 3 times. In the first two trials, the bullet did not hit the exact center of the target. In Trial 1, the bullet landed slightly off-center to the left inside the cup. This was attributed to error in aligning the pendulum with the target, and the target was adjusted slightly. In Trial 2, the bullet was centered in the target but hit the lower inside edge of the cup. This was also attributed to error in aligning the pendulum, which may have been pushed back while loading the spring. Trial 3 resulted in the bullet hitting the inside of the cup in the center.

The average velocity (ignoring uncertainty) was used to calculate the horizontal displacement of the bullet; had the minimum and maximum velocity uncertainties been used, the target would have been within range but not necessarily central to where the bullet landed.

Conclusion

Referring back to the research question, the horizontal displacement required for the bullet shot from the ballistic pendulum to hit the target was 256.7 +/- 0.15 cm. Using the displacement between the photogates and the times the bullet passed through each photogate, the average horizontal velocity was calculated to 602.6 cm/s, uncertainty interval (601.0, 604.1) cm/s. By measuring out the vertical displacement (89.1 +/- 0.1 cm), it was possible to calculate the time it would take for the bullet to land in the cup (for the experiment to end) - a total of 0.464 seconds. The motion of a projectile can be separated into motion along a horizontal and vertical axis; what connects both axes is time. Once the time was found for the bullet falling the length of the vertical axis, the time was found simultaneously for the bullet traveling along the horizontal axis. Using the time calculated along with the average horizontal velocity, the horizontal displacement was derived and the bullet was able to hit the target accurately.

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(-3) But you must calculate uncertainty for time to fall and for dx, using you[r] uncert. for t and fo[r] V.

-3

-3

Appendices

Appendix A

Average horizontal velocity (ignoring uncertainty)

trial	**displacement between photogates (cm)**	**time between photogates (s)**	**average velocity of bullet (ignoring uncertainty) (cm/s)**
1	19.4 +/- 0.05 cm	.032394 +/- 0.000001	598.9
2	19.4 +/- 0.05 cm	.032394 +/- 0.000001	598.9
3	19.4 +/- 0.05 cm	.032019 +/- 0.000001	605.9
4	19.4 +/- 0.05 cm	.032021 +/- 0.000001	605.9
5	19.4 +/- 0.05 cm	.032148 +/- 0.000001	603.5
overall average			602.6

Appendix B

Average horizontal velocity Uncertainty Range

trial	**max displacement (cm)**	**min displacement (cm)**	**max time (s)**	**min time (s)**	**max velocity (cm/s)**	**min velocity (cm/s)**
1	19.45	19.35	.032395	.032393	600.4	597.4
2	19.45	19.35	.032395	.032393	600.4	597.3
3	19.45	19.35	.032020	.032018	607.5	604.3
4	19.45	19.35	.032022	.032020	607.4	604.3
5	19.45	19.35	.032149	.032147	605.0	601.9
overall average					604.1	601.0

Appendix C - Equations

$$V_x = \frac{\Delta x}{t}$$

$$V_{0x} = V_x$$

$$y - y_0 = \frac{1}{2}at^2 + V_{0y}t$$

$$x - x_0 = \frac{1}{2}at^2 + V_{0x}t$$

$$maximum\ uncertainty = \frac{max\ \Delta x}{min\ t}$$

$$minimum\ uncertainty = \frac{min\ \Delta x}{max\ t}$$