A Sphere that's Not Near:
Transition Item #2 of 2

E&M: ASASO I&P

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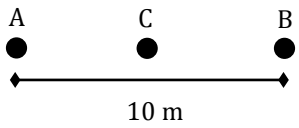
Part 1:
Point Masses: A Review

a) Two point masses (A and B), 5000 kg each, are placed 10 m apart. A third point mass (C) of the same magnitude is placed *directly* between A and B, as shown below.



Calculate the NET gravitational force ON the mass C.

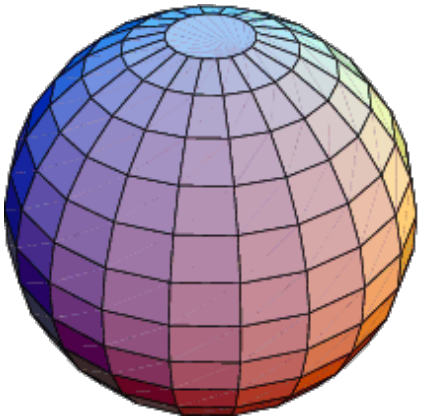
HINT 1: Calculate the gravitational force on mass C due to mass A.
HINT 2: Calculate the gravitational force on mass C due to mass B.
HINT 3: How do you combine forces to get a net force?

b) Same scenario as problem (a), except that mass C is placed 4 m to the right of A (i.e. 6 m to the left of B.) Calculate the NET gravitation force ON mass C.

c) Same scenario as problem (b), but now we're going to change the magnitude of mass B. What mass should we give B so that the net force on C will be ZERO.

Part 2:
Non-Point Masses, *aka* Continuous Masses, *aka* Masses that have *Shapes*

A fictional planet called '*Iamnot*' is measured to have a mass of M, a radius of R and a density (mass/volume) of ρ. Variations in both the planet's radius and density are too small to register on the measuring devices. As far as science is concerned, therefore, *Planet Iamnot* is of essentially ***constant radius*** and ***uniform density***.



You live on a tiny far-flung satellite called '*Luna Staten*'. *Luna Staten* has a mass of ***m*** (not the same as M) but in volume is, indeed, tiny. Compared to *Iamnot*, *Luna Staten* is but a ***particle***—located at a ***point***... in space... and time.

Luna Staten, is a distance ***r*** away from the center of *Iamnot*.
No other measurable bit of mass can be detected for parsecs and parsecs and parsecs...

I. The Theme

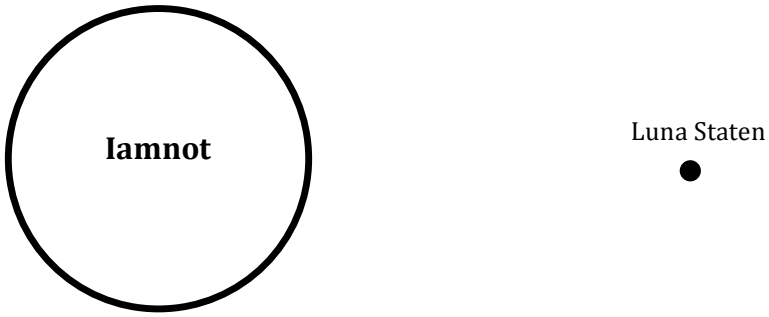
a) Expressed as a function of given and fundamental constants, what is the ***Gravitational Force*** exerted on *Luna Staten* by *Iamnot*?

b) Expressed as a function of given and fundamental constants, what is the ***Gravitational Force*** exerted by *Luna Staten* on *Iamnot*?

c) Expressed as a function of given and fundamental constants, what is the instantaneous rate at which *Luna Staten* will begin to accelerate toward *Iamnot*?

II. The First Variation: Assume that the above facts remain the same EXCEPT:

Iamnot is now a SPHERICAL SHELL. That is, it has the same total mass and radius as described before, but now it consists of pure vacuum--other than an infinitesimally thin and perfectly spherical surface. Funny how things change.



a) Expressed as a function of given and fundamental constants, what is the ***Gravitational Force*** exerted on *Luna Staten* by *Iamnot*?

In order to contemplate this question, think of Iamnot as consisting of zillions and zillions of little point-masses, all arranged in a spherical shell. Each little point of mass pulls on Luna Staten. What is the NET result of all these little pulls?

Think about this until you see why it's complicated and difficult. Explain.

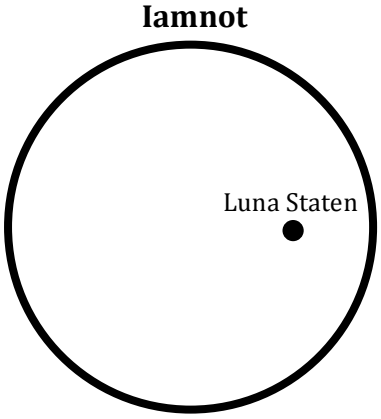
Think even more until you see how it might turn out to be easy to answer even though it's complicated. (The textbook reading might help!)

b) Expressed as a function of given and fundamental constants, what is the ***Gravitational Force*** exerted by *Luna Staten* on *Iamnot*?

c) Expressed as a function of given and fundamental constants, what is the instantaneous rate at which *Luna Staten* will begin to accelerate toward *Iamnot*?

III. The second variation: Assume that none of the above facts change EXCEPT:

Luna Staten is now somewhere inside that SPHERICAL SHELL. That is, $r < R$.



a) Expressed as a function of given and fundamental constants, what is the ***Gravitational Force*** exerted on *Luna Staten* by *Iamnot*?

HINT: Before you try to answer that., consider this simpler question: in the scenario shown in the diagram to the right, which direction will Luna Staten be pulled? To the right? To the left? Up? Down?

As in the preceding variation, think of Iamnot as consisting of zillions and zillions of little point-masses, all arranged in a spherical shell. Each little point of mass pulls on Luna Staten. What is the NET result of all these little pulls?

b) Expressed as a function of given and fundamental constants, what is the ***Gravitational Force*** exerted by *Luna Staten* on *Iamnot*?

c) Expressed as a function of given and fundamental constants, what is the instantaneous rate at which *Luna Staten* will begin to accelerate toward *Iamnot*?